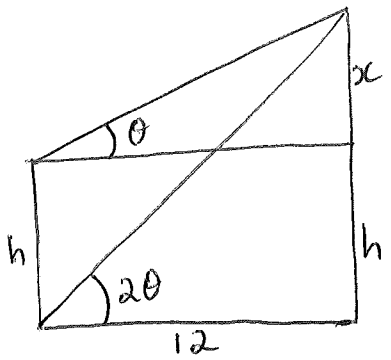
(1)



- Expression for angle θ: $\frac{x}{12} = \tan\theta \Rightarrow x = 12\tan\theta$
- Expression for angle 2θ: $\frac{x+h}{12} = \tan 2\theta$

- $\frac{x+h}{12} = \tan 2\theta$

$x + h = 12\tan 2\theta$

$h = 12\tan 2\theta - x$

$h = 12\tan 2\theta - 12\tan\theta$

$h = 12(\tan 2\theta - \tan\theta)$ ← using $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$h = 12\left(\frac{2\tan\theta}{1-\tan^2\theta} - \tan\theta\right)$

$h = 12\tan\theta\left(\frac{2}{1-\tan^2\theta} - 1\right)$

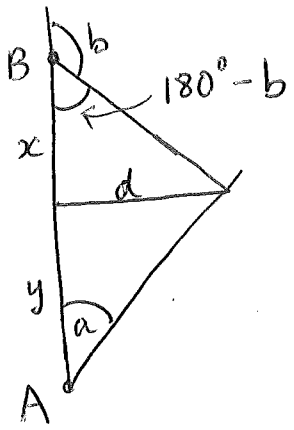
$h = 12\tan\theta\left(\frac{2}{1-\tan^2\theta} - \frac{1-\tan^2\theta}{1-\tan^2\theta}\right)$

$h = 12\tan\theta\left(\frac{2-(1-\tan^2\theta)}{1-\tan^2\theta}\right)$

$h = 12\tan\theta\left(\frac{1+\tan^2\theta}{1-\tan^2\theta}\right)$

$h = \frac{12\tan\theta(1+\tan^2\theta)}{1-\tan^2\theta}$ ✱

(2)



a) • Expression for angle $(180^\circ - b)$: $\frac{d}{x} = \tan(180^\circ - b)$

$$\Rightarrow x = \frac{d}{\tan(180^\circ - b)}$$

• Expression for angle (a) : $\frac{d}{y} = \tan(a)$

$$\Rightarrow y = \frac{d}{\tan(a)}$$

• $AB = x + y$

$$= \frac{d}{\tan(180^\circ - b)} + \frac{d}{\tan(a)}$$

$$= d\left(\frac{1}{\tan(180^\circ - b)} + \frac{1}{\tan(a)}\right)$$

$$= d\left(\frac{1}{\frac{\tan 180^\circ - \tan b}{1 + \tan 180^\circ \tan b}} + \frac{1}{\tan(a)}\right)$$ ↙ using $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$$= d\left(\frac{1 + \tan 180^\circ \tan b}{\tan 180^\circ - \tan b} + \frac{1}{\tan a}\right)$$

$$= d\left(\frac{1}{-\tan b} + \frac{1}{\tan a}\right)$$ ↙ $\tan 180^\circ = 0$

$$= d\left(\frac{1}{\tan a} - \frac{1}{\tan b}\right)$$

$$= d(\cot a - \cot b)$$ ↙ using $\cot x = \frac{1}{\tan x}$

※

(2) continued

b) $AB = d(\cot a - \cot b)$

$= d\left(\frac{\cos a}{\sin a} - \frac{\cos b}{\sin b}\right)$ ← using $\cot x = \frac{\cos x}{\sin x}$

$= d\left(\frac{\cos a \sin b - \sin a \cos b}{\sin a \sin b}\right)$ ← using Product rule to change to sums

$= d\left(\frac{\frac{1}{2}[\sin(a+b) - \sin(a-b)] - \frac{1}{2}[\sin(a+b) + \sin(a-b)]}{\sin a \sin b}\right)$

$= d\left(\frac{\frac{1}{2}\sin(a+b) - \frac{1}{2}\sin(a-b) - \frac{1}{2}\sin(a+b) - \frac{1}{2}\sin(a-b)}{\sin a \sin b}\right)$

$= d\left(\frac{-\sin(a-b)}{\sin a \sin b}\right)$ ← using $\sin(-x) = -\sin(x)$

$= d\left(\frac{\sin(b-a)}{\sin a \sin b}\right)$

$= \frac{d\sin(b-a)}{\sin a \sin b}$ ※